MATH 8600 (FALL 2018) HOMEWORK 6

Assigned 11/01/18, due 11/13/18 in class.

Instructor: Dr. Fei Xue, Martin O-203, fxue@clemson.edu.

Q1-7 Textbook Problems 10.4(a), 10.6, 10.7, 10.13, 10.21, 11.3, 11.5, 11.8.

Q8 Show that the Lagrange basis polynomials satisfy $\sum_{i=0}^{n} L_i(x) \equiv 1$ for all x. (Hint: consider the polynomial interpolation for $f(x) \equiv 1$ at $x_0, x_1, \ldots, x_n$; is $f(x)$ itself a polynomial of degree $\leq n$ going through these data points?)

Q9 **(a)** Consider $f(x) = e^{-5x} \sin \frac{1}{x} \sin \dfrac{1}{\sin \frac{1}{x}}$ on $[a, b] = [0.17, 0.31]$. Use 121 ($n = 120$) equidistant interpolation points $x_0, x_1, \ldots, x_n$ on $[a, b]$, where $x_0 = a$ and $x_n = b$, and construct the Lagrange interpolation $p_n(x)$ in the type-1 barycentric formula. To see whether $p_n(x) \approx f(x)$, take $m = 1024$ random evaluation points $\{s_k\}_{k=1}^{m}$ uniformly distributed on $[0.19, 0.29]$, evaluate $p_n(x)$ and $f(x)$ at $\{s_k\}$ and compute $\|[f(s_1) - p_n(s_1), \ldots, f(s_m) - p_n(s_m)]\|_\infty$. Then, let $\{s_k\}$ be uniformly distributed on $[0.185, 0.295]$ and on $[0.17, 0.31]$, respectively, and repeat. Make comments.

(b) Use Chebyshev interpolation points $x_k = -\cos\left(\frac{k\pi}{n}\right) \frac{b-a}{2} + \frac{b+a}{2}$ and type-2 barycentric formula, generate $m = 1024$ uniformly distributed evaluation points $\{s_k\}_{k=1}^{m}$ on $[a, b]$, repeat the experiments in part (a).

(c) Let $[a, b] = [0.1596, 0.3175]$, and take 16384 random uniformly distributed evaluation points $\{s_k\}$ on $[a, b]$. Evaluate $\|f(\{s_k\}) - p_n(\{s_k\})\|_\infty$ for $n = 600, 800, 900, 1000,$ and 1100, using type-2 barycentric formula.

(d) For the test in part (c), how efficiently would the classical Lagrange formula $p_n(x) = \sum_{i=0}^{n} y_i L_i(x)$ or the type-1 barycentric formula $p_n(x) = L(x) \sum_{i=0}^{n} \frac{\mu_i y_i}{x - x_i}$ work? You may test and time them. (Hint: the arithmetic cost for each formula?)