MATH 8600 (FALL 2018) HOMEWORK 5

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Assigned 10/23/2018, due 10/30/2018 in class.

1. **Prob 8.3, Prob 8.6 (a)(b)** (Prob 8.6(b) is probably challenging)

   (Prob 8.6(b) hint: first, let (λ, v) be an eigenvector of a column-stochastic matrix P, then show that if the sum of all entries of v is nonzero, then $\lambda = 1$; then, assume that $\|v\|_2 = 1$, and show that v^*P is a row vector whose entries are all no greater than $\|v\|_2 = 1$ in modulus (need Cauchy-Schwarz inequality and note that the 2-norm of each column of P is no greater than 1), so that $|v^*Pv| = |\lambda| \leq 1$)

2. Suppose that $X \in \mathbb{R}^{n\times p}$ $(2 \leq p \ll n)$ contains column vectors that are all linear combinations of a particular set of p eigenvectors of $A \in \mathbb{R}^{n\times n}$. Show that the eigenvalues of the block Rayleigh quotient $M = (X^*X)^{-1}(X^*AX)$ are the eigenvalues of A corresponding to these eigenvectors. (Hint: let $X = [v_1, \ldots, v_p]Z$ where Z is a nonsingular matrix of order p)

3. Develop MATLAB code for subspace iteration and shift-invert subspace iteration. Use them to compute the dominant 11 dominant eigenvalues and the 9 eigenvalues closest to $\sigma = -0.1$. Use tolerance 10^{-10} for the relative eigenresidual norm. Plot the relative eigenresidual norm against the iteration number, and compare the computed eigenvalues with the results obtained by using MATLAB's `eigs` function.