1 Functions

1.1 High Level Review

For each of the following questions, give two answers: one for an algebraic definition of a function, e.g. $f(x) = ...$, and another for a graphical definition of a function.

1. How can we determine if a relation is a valid function (well-defined)?
2. What does it mean for a function to be undefined?
3. What is the difference between the domain of a function and the range of a function? How can we find them?
4. What does "evaluate a function at a point" mean?
5. What is a root of function?
6. What does it mean for two functions to be inverses of each other?
7. When does a function have an inverse and when does it not?
8. How can we determine the intersection of two functions?
9. What is a secant line between two points and how can we find it?

1.2 Practice Questions

1. What point(s) is the function $f(x) = \frac{x^2-4}{x+2}$ undefined at if any?
2. Is $f(x) = \begin{cases} 1 & x \geq 0 \\ -1 & x \leq 0 \end{cases}$ a function?
3. Draw a graph of $f(x) = x^2$ and draw the secant line that intersects $x = -2$ and $x = 2$.
4. What is the domain of $f(x) = \log x$?
5. Find the inverse of the function $f(x) = x^3$ and plot the functions on the same graph.
6. Find the inverse of the function $f(x) = e^x$ and state the domain and range of each function.
7. For $f(x) = kx^2 - 6x + 9$, find the value of k such that $f(x)$ has a root at $x = 3$.
8. How many real roots does $f(x) = x^2$ have? What about $g(x) = -x^2$?

2 Limits

2.1 High Level Review

For each of the following questions, give two answers: one for an algebraic definition of a function, e.g. $f(x) = ...$, and another for a graphical definition of a function.

1. What is the difference between a left and right hand limit? How can we evaluate either of them?
2. When does a limit exist and when is it undefined?
3. Can a limit exist even if the function does not exist?
4. What does it mean for a function to be continuous at a point?
5. What does it mean for a function to be continuous over its domain?

2.2 Practice Questions

1. Draw a discontinuous function at $x = 4$.
2. Evaluate $\lim_{x\to 0^+} x^3$ and $\lim_{x\to 0^-} x^3$. Does $\lim_{x\to 0} x^3$ exist?
3. Is $f(x) = e^x$ continuous at $x = \ln 2$?
4. Where is $f(x) = \tan(x)$ not continuous in $[0, 2\pi]$?
5. Draw an example of a function that is defined to be 3 at $x = 1$, but $\lim_{x\to 1} f(x) = 2$. Is this function continuous at $x = 1$?

3 Derivatives

3.1 High Level Review

For each of the following questions, give two answers: one for an algebraic definition of a function, e.g. $f(x) = ...$, and another for a graphical definition of a function.

1. When does a derivative of a function $f(x)$ exist?
2. What does a derivative represent?
3. How are slope and a derivative related?
4. What is the difference between a secant line and a tangent line?
5. What is a cusp?

3.2 Practice Questions

1. Evaluate $\lim_{h \to 0} \frac{e^4(e^h - 1)}{h}$.
2. Plot a graph of $f(x) = \sin(x)$ and determine all intervals in $[0, 2\pi]$ such that $f(x)$ is decreasing.
3. Let $f(t) = 3t^2 - 20t$ represent my speed over a run. At what time t have I stopped moving?
4. Let $f(x) = -x^2 + 9$. Find the slope of the tangent line that intersects $x = -3$ and $x = 3$. Compare this to the slope of the tangent line at $x = 0$.
5. Where is $f(x) = |x|$ not differentiable?